



Key

UNIT 9 - SOLVING RADICAL EQUATIONS



What is the opposite of taking the square root of a number? Square the #

$$\sqrt{4} = 2 \quad (2)^2 = 4$$

Solve: $\sqrt{x+8} + 3 = 10$

Step 1: Isolate the radical expression:

$$\begin{array}{r} \sqrt{x+8} + 3 = 10 \\ \quad \quad -3 \quad -3 \\ \hline \end{array}$$

$$\begin{array}{l} \sqrt{x+8} + 3 = 10 \\ \sqrt{4+8} + 3 = 10 \\ \sqrt{49} + 3 = 10 \\ 7 + 3 = 10 \\ \boxed{10 = 10} \\ \text{☺} \end{array}$$

$$(\sqrt{x+8})^2 = (-7)^2$$

Step 2: Square both sides of the equation:

$$\begin{array}{r} x+8 = 49 \\ \quad -8 \quad -8 \\ \hline \end{array}$$

$$\boxed{x=41}$$

Step 3: Solve the resulting equation:

Step 4: Check answers:

Solve & Check each radical equation:

a) $\sqrt{x} - 10 = 0$

$$\begin{array}{r} \sqrt{x} - 10 = 0 \\ \quad +10 \quad +10 \\ \hline (\sqrt{x})^2 = (10)^2 \end{array}$$

$$x = 100$$

$$\sqrt{x} - 10 = 0$$

$$\sqrt{100} - 10 = 0$$

$$10 - 10 = 0$$

$$0 = 0$$

yes ☺

b) $\frac{8\sqrt{x-3}}{8} = \frac{16}{8}$

$$(\sqrt{x-3})^2 = (2)^2$$

$$x-3 = 4$$

$$\quad +3 \quad +3$$

$$\boxed{x=7}$$

$$8\sqrt{x-3} = 16$$

$$8\sqrt{7-3} = 16$$

$$8\sqrt{4} = 16$$

$$8 \cdot 2 = 16$$

$$16 = 16$$

$$16 = 16$$

$$16 = 16$$

yes ☺

(on own)
(so over)

(with partner)

(celebrity)

$$\begin{aligned} \sqrt{3x+1}-2 &= 6 \\ +2 \quad +2 \\ \hline (\sqrt{3x+1})^2 &= (8)^2 \\ 3x+1 &= 64 \\ 3x &= 63 \\ \boxed{x=21} \end{aligned}$$

$$\begin{aligned} \sqrt{3x+1}-2 &= 6 \\ \sqrt{3 \cdot 21+1}-2 &= 6 \\ \sqrt{63+1}-2 &= 6 \\ \sqrt{64}-2 &= 6 \\ 8-2 &= 6 \\ 6 &= 6 \\ \text{yes} \text{ 😊} \end{aligned}$$

$$\begin{aligned} d) (\sqrt{x-6})^2 &= (4)^2 \\ x-6 &= 16 \\ +6 \quad +6 \\ \hline x &= 22 \end{aligned}$$

$$\begin{aligned} \sqrt{x-6} &= 4 \\ \sqrt{22-6} &= 4 \\ \sqrt{16} &= 4 \\ 4 &= 4 \\ \text{yes} \text{ 😊} \end{aligned}$$



(celebrity)

$$\begin{aligned} e) (\sqrt{y+1})^2 &= (1)^2 \\ y+1 &= 1 \\ \boxed{y=0} \end{aligned}$$

$$\begin{aligned} \sqrt{y+1} &= 1 \\ \sqrt{0+1} &= 1 \\ \sqrt{1} &= 1 \\ 1 &= 1 \\ \text{yes} \text{ 😊} \end{aligned}$$

$$\begin{aligned} f) \sqrt{x-4}+5 &= 11 \\ (\sqrt{x-4})^2 &= (6)^2 \\ x-4 &= 36 \\ \boxed{x=40} \end{aligned}$$

$$\begin{aligned} \sqrt{x-4}+5 &= 11 \\ \sqrt{40-4}+5 &= 11 \\ \sqrt{36}+5 &= 11 \\ 6+5 &= 11 \\ 11 &= 11 \\ \text{yes} \text{ 😊} \end{aligned}$$

(on own)

$$\begin{aligned} g) 2\sqrt{3y+1}-3 &= 1 \\ \frac{2\sqrt{3y+1}}{2} &= \frac{4}{2} \\ (\sqrt{3y+1})^2 &= (2)^2 \\ 3y+1 &= 4 \\ 3y &= 3 \\ \boxed{y=1} \end{aligned}$$

$$\begin{aligned} 2\sqrt{3y+1}-3 &= 1 \\ 2\sqrt{3 \cdot 1+1}-3 &= 1 \\ 2\sqrt{3+1}-3 &= 1 \\ 2\sqrt{4}-3 &= 1 \\ 2 \cdot 2-3 &= 1 \\ 4-3 &= 1 \\ 1 &= 1 \\ \text{yes} \text{ 😊} \end{aligned}$$

EXTRANEOUS SOLUTIONS

When you solve an equation by squaring each side, you create a new equation. This new equation may have solutions that do not solve the original equation.

| Original equation | Square each side | New equation | Solutions of new equation |
|-------------------|------------------|--------------|---------------------------|
| $(x) = (2)$ | $x^2 = 4$ | $x^2 = 4$ | $x = \pm 2$ |



Extraneous Solution:

Use those steps to solve and see what happens:

| | | |
|------------------------|----------------------|------------------------|
| $(\sqrt{2x+15}) = (x)$ | $\sqrt{2(5)+15} = 5$ | $\sqrt{2x+15} = x$ |
| $2x+15 = x^2$ | $\sqrt{25} = 5$ | $\sqrt{2(-3)+15} = -3$ |
| $0 = x^2 - 2x - 15$ | $5 = 5$ | $\sqrt{-6+15} = -3$ |
| $(x-5)(x+3)$ | yes 😊 | $\sqrt{9} = -3$ |
| $x = 5 \quad x = -3$ | | $3 = -3$ |
| | | No 😞 |

Try on your own!

| | | | | | |
|--|--------------------|----------------|----------------------|------------------|-------------------|
| 1. $x = \sqrt{12-x}$ | $-4 = \sqrt{12-4}$ | $3 = \sqrt{9}$ | 2. $x = \sqrt{3+2x}$ | $3 = \sqrt{3+6}$ | $-1 = \sqrt{3-1}$ |
| $x^2 = 12-x$ | $-4 = \sqrt{8}$ | $3 = 3$ | $x^2 = 3+2x$ | $3 = \sqrt{9}$ | $-1 = 1$ |
| $x^2 + x - 12 = 0$ | $-4 = 4$ | 😊 | $x^2 - 2x - 3 = 0$ | $3 = 3$ | NO |
| $(x+4)(x-3) = 0$ | 😞 | | $(x-3)(x+1) = 0$ | | |
| $x = -4$ $x = 3$ | | | $x = 3 \quad x = -1$ | | |